Department of Mathematics and Computer Science

Friday, September 22, 2017, 4:10 pm

COLLOQUIUM TALK

Speaker: Gregory Galperin (EIU)

Old Main 2231

Jumping Electrons and Jumping Spheres

Abstract:

In the series of **two EUI Friday Colloquiums** (September 22 & 29, 2017), I will formulate and solve two math problems, solutions of which are based on the idea of applying a tricky function to the system configurations. The titles of the talks are "**Jumping Electrons**" and "**Jumping Spheres**." Both talks will be quite accessible to a general audience, especially for students.

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In my first talk "**Jumping Electrons**", I will solve the following problem. Consider an "atom" with a fixed number of electrons rotating along different orbits around the nucleus. At some moment, one electron from each orbit jumps out from its orbit and all the jumped electrons form a new orbit around the nucleus. Thus, we get a new state of the atom. (The more electrons on the orbit, the further this orbit is situated from the nucleus.) Then the process iterates infinitely many times. The question is to determine the **final state**(s) of the atom, i.e. to determine the number of orbits in the atom and the number of electrons on each orbit in its final state.

For example, if the total number of electrons is $N = 10$ and they split in two orbits, with 6 and 4 electrons, respectively, then the dynamics of the atom will be as follows: $(6, 4) \to (5, 3, 2) \to (4, 3, 2, 1) \to (4, 3, 2, 1) \to ...$, that is, the final atom's state is the 4-tuple $(4, 3, 2, 1)$. It turns out that **an arbitrary initial state** of atom with 10 electrons will lead to the same final state $(4, 3, 2, 1)$! I will prove this result for all *triangular numbers* N, the ones that can be represented as the sum of consecutive integers starting with the 1.

Homework: Consider $N = 15$ electrons and the 3-orbit atom's state $(8, 5, 2)$.

For *non-triangular* values of N, there will be more than one final state. All such final states form a cycle which depends not only on the total number of electrons N, but also on the initial state of the atom (i.e. on the number of orbits and the number of electrons on these orbits).

Though the problem sounds purely combinatorial, my solution of it is based on the atom's "**potential energy**", the term borrowed from physics.

SNACKS IN FACULTY LOUNGE AT 3:30 PM.
EVERYONE WELCOME (EVEN IF YOU ARE UNABLE TO ATTEND THE TALK)